Level – 2	**Daily Tutorial Sheet - 11**

131.(B) $I = \int \sqrt{\frac{x^6+1}{x^6}} \log \frac{x^6+1}{x^6} \cdot \frac{1}{x^7} dx$ $\left[\text{Write } \frac{1}{x^{10}} = \frac{1}{x^7 \cdot x^3} = \frac{1}{x^7 \cdot \sqrt{x^6}}\right]$

Put $\frac{x^6+1}{x^6} = 1 + \frac{1}{x^6} = t \quad \Rightarrow \quad \frac{-6}{x^7} dx = dt$

$I = \int -\frac{1}{6}\sqrt{t} \log t \, dt = \frac{-1}{6}\left[\frac{2}{3} t^{3/2} \log t - \frac{2}{3}\int t^{3/2} \frac{1}{t} dt\right] = -\frac{1}{6}\left[\frac{2}{3} t^{3/2} \log t - \frac{4}{3} t^{3/2}\right] + C$ $\left[\text{when } t = 1 + \frac{1}{x^6}\right]$

132.(C) $I = \int e^x \left[\frac{1-x^{2n}}{(1-x)^n \sqrt{1-x^{2n}}} + \frac{nx^{n-1}}{\left(1-x^n\right)\sqrt{1-x^{2n}}}\right] dx$

Now $\frac{\sqrt{1-x^{2n}}}{1-x^n} = \frac{\sqrt{\left(1-x^n\right)\left(1+x^n\right)}}{1-x^n} = \sqrt{\frac{1+x^n}{1-x^n}}$ and $\left[\frac{d}{dx}\left(\sqrt{\frac{1+x^n}{1-x^n}}\right) = \frac{nx^{n-1}}{\left(1-x^n\right)\sqrt{1-x^{2n}}}\right]$

$I = e^x \sqrt{\frac{1+x^n}{1-x^n}}$

133.(A) Put $x^5 = t \qquad \therefore \qquad I = \frac{1}{5}\int \frac{t^2-1}{t^4+3t^2+1} dt = \frac{1}{5}\tan^{-1}\left(t + \frac{1}{t}\right)$

134.(B) $I = \int \frac{\sin x \cdot \sqrt{\cos x}}{\sqrt{\left\{1-\left(\cos^{3/2} x\right)^2\right\}}}$ [Put $\cos^{3/2} x = t$]

$\Rightarrow \qquad I = \frac{2}{3}\sin^{-1}\left(\cos^{3/2} x\right)$

135.(B) Let $I = \int \frac{\sin^{1/2} x}{\cos^{9/2} x} dx = \int \frac{dx}{\sin^{-1/2} x \cos^{9/2} x}$; Here $m + n = \frac{1}{2} - \frac{9}{2} = -4$ (negative even integer).

Divide numerator and denominator by $\cos^4 x$.

$I = \int \sqrt{\tan x} \sec^4 x \, dx = \int \sqrt{\tan x} \left(1+\tan^2 x\right)\sec^2 x \, dx = \int \sqrt{t}\left(1+t^2\right) dt$ (Using $\tan x = t$)

$= \frac{2}{3} t^{3/2} + \frac{2}{7} t^{7/2} + c = \frac{2}{3}\tan^{3/2} x + \frac{2}{7}\tan^{7/2} x + c$

136.(B) $I = \int \frac{\cos^4 x}{\sin^3 x \left(\sin^5 x + \cos^5 x\right)^{3/5}} dx = \int \frac{\cos^4 x}{\sin^6 x \left(1+\cot^5 x\right)^{3/5}} dx = \int \frac{\cot^4 x \, \operatorname{cosec}^4 x \, dx}{\left(1+\cot^5 x\right)^{3/5}}$

$= \frac{1}{5}\int \frac{dt}{t^{3/5}} = -\frac{1}{2} t^{2/5} + c = -\frac{1}{2}\left(1+\cot^5 x\right)^{2/5} + c$ (Put $1+\cot^5 x = t \;\Rightarrow\; 5\cot^4 x \operatorname{cosec}^2 x \, dx = -dt$)

137.(A) Here, $I = \int \sqrt{\frac{3-x}{3+x}} \, . \sin^{-1}\left(\frac{1}{\sqrt{6}}\sqrt{3-x}\right) dx$ (Put $x = 3\cos 2\theta \Rightarrow dx = -6\sin 2\theta \, d\theta$)

$= \int \sqrt{\frac{3-3\cos 2\theta}{3+3\cos 2\theta}} \, . \sin^{-1}\left(\frac{1}{\sqrt{6}}\sqrt{3-3\cos 2\theta}\right)\left(-6\sin 2\theta\right) d\theta$

$$= \int \frac{\sin\theta}{\cos\theta} \,.\, \sin^{-1}(\sin\theta)\, d\theta(-6\sin 2\theta)\, d\theta = -6\int \theta\left(2\sin^2\theta\right) d\theta = -6\left\{\frac{\theta^2}{2} - \int \theta\cos 2\theta\, d\theta\right\}$$

$$= -6\left\{\frac{\theta^2}{2} - \left(\theta\frac{\sin 2\theta}{2} - \int 1 \,.\, \left(\frac{\sin 2\theta}{2}\right) d\theta\right)\right\} = -3\theta^2 + 6\left\{\theta\frac{\sin 2\theta}{2} + \frac{\cos 2\theta}{4}\right\} + c$$

$$= \frac{1}{4}\left\{-3\left(\cos^{-1}\left(\frac{x}{3}\right)\right)^2 + 2\sqrt{9-x^2} \,.\, \cos^{-1}\left(\frac{x}{3}\right) + 2x\right\} + c$$

138.(D) $I = \int \dfrac{\tan\left(\dfrac{\pi}{4} - x\right)}{\cos^2 x\sqrt{\tan^3 x + \tan^2 x + \tan x}}\, dx = \int \dfrac{\left(1-\tan^2 x\right) dx}{(1+\tan x)^2 \cos^2 x\sqrt{\tan^3 x + \tan^2 x + \tan x}}$

$$I = \frac{-\left(1 - \dfrac{1}{\tan^2 x}\right)\sec^2 x\, dx}{\left(\tan x + 2 + \dfrac{1}{\tan x}\right)\sqrt{\tan x + 1 + \dfrac{1}{\tan x}}} \text{ Let, } y = \sqrt{\tan x + 1 + \frac{1}{\tan x}} \Rightarrow 2y\; dy = \left(\sec^2 x - \frac{1}{\tan^2 x} \,.\, \sec^2 x\right) dx$$

$$\therefore \qquad I = \int \frac{-2y\, dy}{\left(y^2+1\right) . \,y} = -2\int \frac{dy}{1+y^2} = -2\tan^{-1} y + c = -2\tan^{-1}\left(\sqrt{\tan x + 1 + \frac{1}{\tan x}}\right) + c$$

139.(B) $\displaystyle\int \frac{x^{1/3}}{\left(x^4-1\right)^{4/3}}\, dx = \frac{1}{4}\int \frac{4x^{-5}}{\left(1-x^{-4}\right)^{4/3}}\, dx = \frac{-3}{4}\left(1-x^{-4}\right)^{-1/3} + C$ (Put $1 - x^{-4} = t$)

140.(A) Let $1 + \ell n\left|x\right| = t^2$

$$\frac{dx}{|x|} = 2t\, dt = 2\int \frac{\left(t^2-1\right) dt}{t} = 2\left[\frac{t^3}{3} - t\right] + c \text{ (where } t = \sqrt{1+\ell n\left|x\right|}\text{)}$$